Int. J. Heat Mass Transfer. Vol. 24, No. 6, pp. 1059–1069, 1981
Printed in Great Britain

0017-9310/81/061059-11 $02.00/0
Pergamon Press Ltd.

HEAT TRANSFER TO NON-NEWTONIAN AND DRAG-REDUCING FLUIDS IN TURBULENT PIPE FLOW

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(*Received* 3 *September* 1980 *and in revised form* 5 *December* 1980)

Abstract—A theoretical approach based upon the use of Newtonian eddy viscosity correlations has been applied to the calculation of heat transfer coefficients for non-Newtonian and drag-reducing liquids in fully-developed turbulent pipe flow. Previously published data of Gupta [19] and Friend [21], over a Reynolds number range of 6×10^3–1.5×10^5, have been compared with the predicted values and good agreement has been obtained.

NOMENCLATURE

A, B, constants in the logarithmic law of the wall;
C_1, C_2, constants in eddy viscosity expressions;
C_f, Fanning friction factor $(2\tau_w/\rho\langle v_z\rangle^2)$;
C_h, the Stanton number $(h/\rho C_p\langle v_z\rangle)$, dimensionless heat transfer coefficient;
C_p, heat capacity [$\mathrm{J\,kg^{-1}\,K^{-1}}$];
D, pipe diameter [m];
h, heat transfer coefficient [$\mathrm{W\,m^{-2}\,K^{-1}}$];
k, thermal conductivity [$\mathrm{W\,m^{-1}\,K^{-1}}$];
K, parameter in power-law expression for non-Newtonian viscous properties [$\mathrm{kg\,m^{-1}\,s^{n-2}}$];
n, index in power-law expression;
Nu, Nusselt number (hD/k);
Pr, Prandtl number $(\mu C_p/k)$;
Pr_{tr}, turbulent Prandtl number $(\varepsilon_{rz}/\varepsilon_{hr})$;
q_r, radial heat flux [$\mathrm{W\,m^{-2}}$];
q_w, wall heat flux [$\mathrm{W\,m^{-2}}$];
r, radial distance [m];
R, pipe radius [m];
R^+, dimensionless radius $(\rho R v^*/\mu)$;
Re, Reynolds number $(\rho\langle v_z\rangle D/\mu)$;
T, temperature [K];
$\bar{T}^+$, dimensionless temperature $\bar{T} - T_w/(q_w/\rho C_p v^*)$;
v_z, velocity [$\mathrm{m\,s^{-1}}$];
v_z^+, dimensionless velocity (v_z/v^*);
$\langle v_z\rangle$, mean velocity [$\mathrm{m\,s^{-1}}$];
v^*, friction velocity [$\sqrt{(\tau_w/\rho)}$] [$\mathrm{m\,s^{-1}}$];
y, distance from pipe wall, m;
y^+, dimensionless distance $(\rho y v^*/\mu)$;
z, axial distance [m].

Greek letters

α, constant in logarithmic temperature profile;
β, constant in temperature correction of Kader and Yaglom [11];
β_1, constant in logarithmic temperature profile;
$\dot{\gamma}$, shear rate [$\mathrm{s^{-1}}$];
ε_{rz}, eddy viscosity [$\mathrm{m^2\,s^{-1}}$];
ε_{hr}, eddy diffusivity for heat [$\mathrm{m^2\,s^{-1}}$];
μ, viscosity [$\mathrm{kg\,m^{-1}\,s^{-1}}$];
ρ, density [$\mathrm{kg\,m^{-3}}$];
τ_{rz}, shear stress [$\mathrm{kg\,m^{-1}\,s^{-2}}$];
τ_w, wall shear stress [$\mathrm{kg\,m^{-1}\,s^{-2}}$].

Subscripts

b, bulk value;
0, centreline value;
q, dummy integration variable;
w, at the pipe wall.

Superscripts

$-$, time average variable.

INTRODUCTION

MOST flows of engineering importance are turbulent and viscosities must be high before laminar flow predominates. When viscosities are high the fluids are often non-Newtonian in character. Thus, in the field of non-Newtonian flow laminar flow tends to predominate. However, there are still many instances in which turbulent flow of non-Newtonian fluids is encountered. In addition, there is an increasing use of high-polymeric additives to achieve reductions in frictional drag. The present paper will deal with heat transfer to both non-Newtonian and drag-reducing fluids in turbulent pipe flow.

Both slurries of approximately spherical particles and polymer solutions can, under certain circumstances, flow turbulently without exhibiting anomalous wall effects. The effects in question are wall 'slip' with slurries and drag-reduction with polymers. We have previously considered [1] the flow of non-Newtonian fluids in the absence of anomalous wall effects and it was concluded that mean velocity profiles and friction factors for Newtonian and non-Newtonian fluids can be represented on the same basis if we replace the Newtonian viscosity by the apparent viscosity at the wall.

Numerous studies have been undertaken to charac-

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terize drag reduction phenomena in polymer solutions. The fact that the addition of very small concentrations of dissolved high polymeric substances can reduce frictional resistance in turbulent flow to as low as one quarter of that of the pure solvent makes the phenomenon potentially very important. The effect of additives on fluid friction has been the subject of reviews by Hoyt [2], Lumley [3] and Virk [4].

Drag-reduction is illustrated in Fig. 1. A phenomenon closely related to drag-reduction, transition delay, is illustrated in Fig. 2. The behaviour shown in Fig. 2(a) is typical of soap solutions [5] and that in Fig. 2(b) typical of certain types of polymer solutions, such as polyacrylamide in water [6]. The distinction between the two phenomena is that with drag-reduction the flow attains non-drag-reducing fully-developed turbulent flow before it is affected by the polymer. Drag reduction and transition delay are no doubt related but, on the basis of the available evidence, there appear to be significant differences. Transition delay will not be considered in this work and only the phenomena illustrated by Fig. 1 will be treated.

We have previously demonstrated [7] how Newtonian eddy viscosity expressions can be applied to drag-reducing flows. In that work, eddy viscosity expressions were used to predict velocity profiles which were in good agreement with experimental measurements.

In this paper, the methods outlined previously [1, 7] for extending Newtonian correlations to non-Newtonian and drag-reducing flows will be extended to consider non-isothermal flows for the prediction of heat transfer coefficients.

THE INTEGRATION OF THE ENERGY EQUATION

Consider axisymmetric pipe flow of an incompressible fluid with a fully-developed velocity profile. Neglecting axial heat conduction and viscous dissipation the equation of energy is

$$\rho C_p \bar{v}_z \frac{\partial \bar{T}}{\partial z} = -\frac{1}{r}\frac{\partial}{\partial r}(r\bar{q}_{r\,\mathrm{eff}}) \tag{1}$$

where $q_{r\,\mathrm{eff}}$ is the effective radial heat flux given by

$$\bar{q}_{r\,\mathrm{eff}} = -(k + \rho C_p \varepsilon_{hr})\frac{\partial \bar{T}}{\partial r}. \tag{2}$$

The eddy diffusivity ε_{hr} in equation (2) can be replaced by the relationship

$$\varepsilon_{hr} = \frac{\varepsilon_{rz}}{Pr_{tr}} \tag{3}$$

where ε_{rz} is the eddy viscosity and Pr_{tr} the turbulent Prandtl number.

For fully-developed flows equation (1) has been integrated for the case of constant wall flux by Lyon

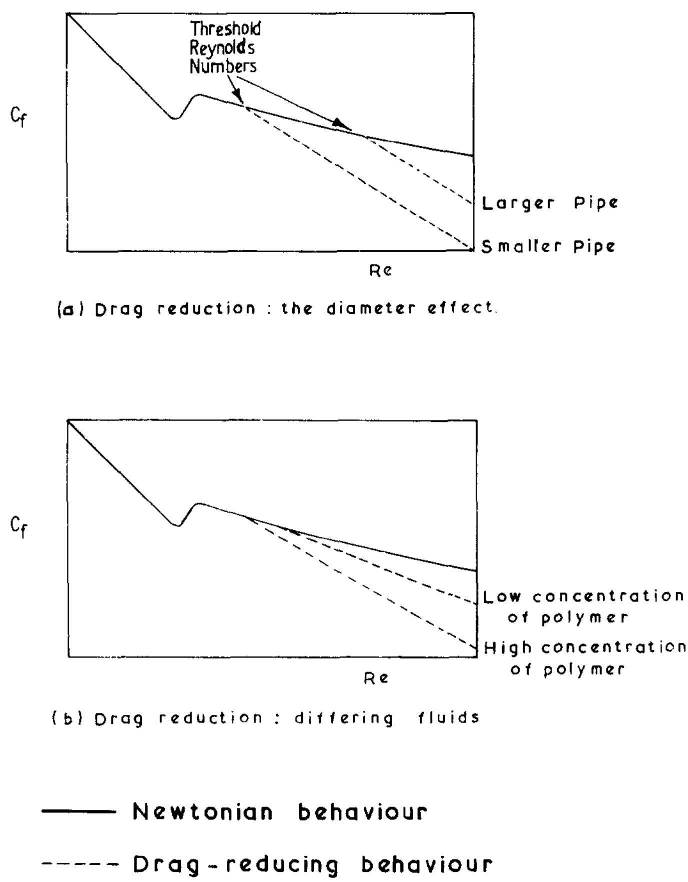

FIG. 1. Drag reduction phenomena.

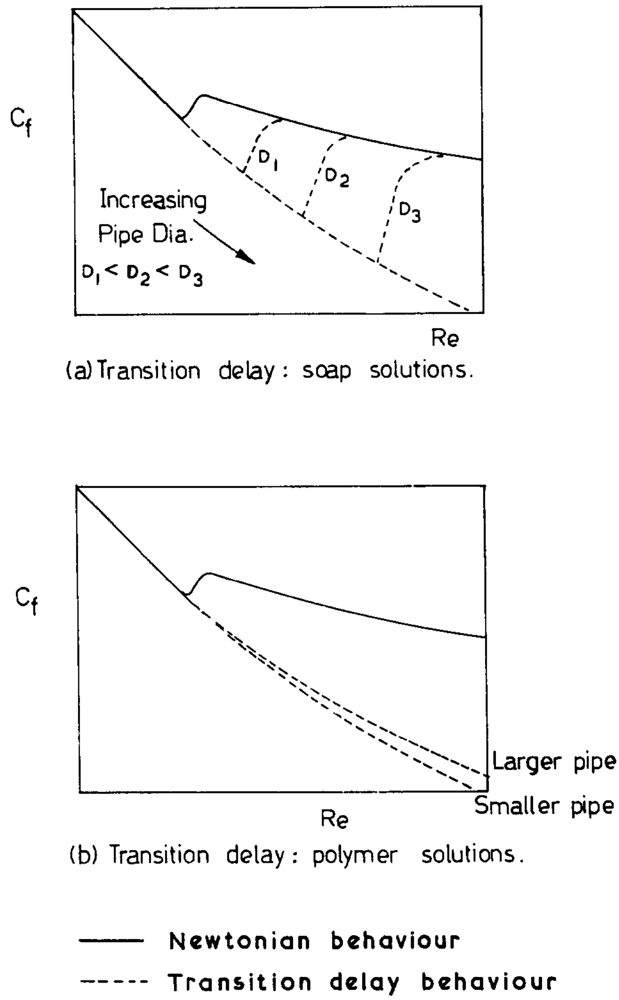

FIG. 2. Transition delay phenomena.

[8] and for the case of constant wall temperature by Seban and Shimazaki [9]. The expressions developed by Lyon and by Seban and Shimazaki are very complex and require the solution of double integrals. However, simplified integrations of equation (1) have been presented previously [10] assuming that the radial heat flux across the pipe varies linearly with radial position. Two expressions for the dimensionless heat transfer coefficient can be developed. The first involves integration with respect to dimensionless distance y^+ [10], i.e.

$$C_h = \sqrt{(C_f/2)} \Bigg/ \int_0^{R^+} \left[(1 - y^+/R^+)\mathrm{d}y^+ \Bigg/ \left(\frac{1}{Pr} + \frac{1}{Pr_{tr}} \frac{\rho \varepsilon_{rz}}{\mu} \right) \right] - \beta. \quad (4)$$

The second expression involves integration with respect to the dimensionless velocity, v_z^+, i.e.

$$C_h = \sqrt{(C_f/2)} \Bigg/ (Pr - Pr_{tr}) \int_0^{v_{z0}^+} \left[\mathrm{d}\bar{v}_z^+ \Bigg/ \left(1 + \frac{Pr}{Pr_{tr}} \frac{\rho \varepsilon_{rz}}{\mu} \right) \right] + \frac{Pr_{tr}}{\sqrt{(C_f/2)}} [4.07\sqrt{(C_f/2)} + 1] - \beta \quad (5)$$

and is derived in the Appendix.

In equations (4) and (5), β is the temperature correction factor of Kader and Yaglom [11] which is discussed in the Appendix.

The turbulent Prandtl number Pr_{tr} has been considered previously [28] and it was concluded that, in the light of the confusing and contradictory experimental evidence, it is reasonable to adopt a value of Pr_{tr} equal to unity, which is the value used in this work.

Before solving either equation (4) or (5) it is necessary to define the eddy viscosity, ε_{rz}. This is considered below.

EDDY VISCOSITY EXPRESSIONS FOR NON-NEWTONIAN AND DRAG-REDUCING FLUIDS

In a previous publication [7] we demonstrated how four standard Newtonian eddy viscosity expressions could be successfully adapted to predict velocity profiles in drag-reducing fluids. These expressions were due to Spalding [12], Wasan, Tien and Wilke [13], Mizushina and Ogino [14] and van Driest [15]. The forms of the various expressions are as follows.

(1) *Spalding* [12]

Spalding suggested an exponential function for the eddy viscosity which decayed to the wall as the cube of the distance, i.e.

$$\frac{\rho\varepsilon_{rz}}{\mu} = \frac{1}{A}\exp\left(-\frac{B}{A}\right)\left[\exp\left(\frac{\bar{v}_z^+}{A}\right) - 1 - \left(\frac{\bar{v}_z^+}{A}\right) - \frac{1}{2}\left(\frac{\bar{v}_z^+}{A}\right)^2\right]. \quad (6)$$

This expression predicts a mean velocity profile which at high values of y^+ approaches asymptotically the logarithmic law of the wall, i.e.

$$\bar{v}_z^+ = A \ln y^+ + B. \quad (7)$$

(2) *Wasan, Tien and Wilke* [13]

These authors proposed an expression based on a series expansion of the mean velocity profile in the near-wall region, i.e.

$$\frac{\rho\varepsilon_{rz}}{\mu} = \frac{-4C_1 y^{+3} - 5C_2 y^{+4}}{1 + 4C_1 y^{+3} + 5C_2 y^{+4}} \quad (8)$$

where C_1 and C_2 are constants which are determined by matching the values of $\bar{v}_z^+$ and the first and second derivatives of $\bar{v}_z^+$ predicted by equation (8) with the corresponding values given by the logarithmic law of the wall, equation (7). The details of the computation of C_1 and C_2 have been published previously [7, 28].

(3) *Mizushina and Ogino* [14]

Mizushina and Ogino divided the pipe radius into three zones and proposed an eddy viscosity expression for each, i.e.

$$\frac{\rho\varepsilon_{rz}}{\mu} = by^{+3} \quad \text{for } 0 \le y^+ \le y_1^+ \quad (9)$$

$$\frac{\rho\varepsilon_{rz}}{\mu} = \frac{y^+}{A}\left(1 - \frac{y^+}{A}\right) - 1 \quad \text{for } y_1^+ \le y^+ \le y_2^+ \quad (10)$$

$$\frac{\rho\varepsilon_{rz}}{\mu} = 0.07R^+ \quad \text{for } y_2^+ \leqslant y^+ \leqslant R^+. \quad (11)$$

To determine the values of b, y_1^+ and y_2^+ it is necessary to impose the restrictions that $\bar{v}_z^+$ and the first derivative of $\bar{v}_z^+$ should be continuous. The constants b and y_1^+ must be computed by an iterative method, the details of which are given elsewhere [7, 14]. The constant y_2^+ is given by

$$y_2^+ = \frac{R^+ - [R^{+2}(1 - 0.28A) - 4AR^+]^{1/2}}{2}. \quad (12)$$

(4) *Van Driest* [15]

Van Driest used a mixing length approach to develop an eddy visocity expression for the wall region, i.e.

$$\frac{\rho\varepsilon_{rz}}{\mu} = \left(\frac{y}{A}\right)^2\left[1 - \exp\left(-\frac{y^+}{C_1^+}\right)^2\right]\frac{\partial\bar{v}_z^+}{\partial y^+} \quad (13)$$

where

$$\frac{\partial\bar{v}_z^+}{\partial y^+} = 2\Bigg/1 + \sqrt{\left\{1 + 4\left(\frac{y^+}{A}\right)^2\left[1 - \exp\left(-\frac{y^+}{C_1^+}\right)\right]^2\right\}}. \quad (14)$$

The equations necessary to calculate the constant C_1^+ have again been given previously [7].

The above expressions for the eddy viscosity can be applied to non-Newtonian fluids by replacing the Newtonian viscosity expression in equations (6)–(14) by the apparent viscosity at the wall [1, 7]. They can also be applied to drag-reducing fluids by assuming

(a) that when drag-reducing velocity profiles are plotted on a law of the wall basis, a logarithmic region is always evident,
(b) that the gradient of the logarithmic region is constant, i.e. the coefficient A in equations (6)–(14) is constant and equal to its Newtonian value of 2.5 and the coefficient B varies with the degree of drag reduction.

These assumptions are supported by the evidence of Elata, Letver and Kabanovitz [16], Arunachalem, Hummel and Smith [17] and Rollin and Seyer [18].

In this paper two methods will be compared for the calculation of the coefficient B. The first is our method [7], i.e.

$$B = \sqrt{(2/C_{f\,DR})} - 2.46 \ln[Re\sqrt{(C_{f\,DR})}] + 5.67 \quad (15a)$$

$$= \frac{\langle\bar{v}_z\rangle}{v^*} - 2.46 \ln\left[\frac{v^* Re}{\langle\bar{v}_z\rangle}\right] + 4.82 \quad (15b)$$

and the other due to Arunachalem, Hummel and Smith [17], i.e.

$$B = B_N\left(\frac{v_N^*}{v^*}\right)^3 \quad (16)$$

where the subscript N refers to the Newtonian values.

The use of the above eddy viscosity expressions in calculating dimensionless heat transfer coefficients using equations (4) or (5) is considered below.

THE CALCULATION OF DIMENSIONLESS HEAT TRANSFER COEFFICIENTS FOR NON-NEWTONIAN AND DRAG-REDUCING FLUIDS IN TURBULENT PIPE FLOW

The starting point for the calculation of dimensionless heat transfer coefficients is the evaluation of the coefficient B from pressure drop data in the turbulent region using either equation (15) or (16). This study uses a comparison between experimental and predicted heat transfer coefficients using experimental measurements of pressure drop. However, if this method were to be used in design calculations, the pressure drop (or friction factor) must be somehow correlated against flowrate. This problem of correlation has been discussed elsewhere [7]. For design work it would be necessary to carry out 'turbulent viscometric' measurements, in addition to the normal laminar flow viscometric measurements required to characterize the fluid, and the heat transfer coefficient calculated from a scaled turbulent pressure drop.

The procedure to calculate a heat transfer coefficient is as follows:

(1) From experimental pressure drop data in a tube of the same diameter as that used for heat transfer, calculate the coefficient B from either equation (15) or (16).

(2) Calculate R^+ from the wall shear stress (i.e. pressure drop) and y_2^+ from equation (12).

(3) (a) *Correlation* [12]. Calculate the values of $\bar{v}_z^+$ corresponding to y_2^+ and R^+ from Spalding's expression for the mean velocity [7]. The integral in equation (5) is solved numerically from 0 to $\bar{v}_{z2}^+$ using equation (6) and from v_{z2}^+ to $\bar{v}_{zR}^+$ using equation (11). Having evaluated the integral then the dimensionless heat transfer coefficient C_h can be calculated directly from equation (5).

(b) *Correlations of* [13–15]. The values of the constants in the various expressions must be evaluated by iterative procedures detailed elsewhere [7]. Each expression requires the determination of y_1^+. In order to evaluate the integral in equation (4), it is divided into three zones 0 to y_1^+, y_1^+ to y_2^+ and y_2^+ to R^+. This integration is obtained numerically and the dimensionless heat transfer coefficient calculated directly from equation (4).

A summary of the equations used is given in Table 1.

COMPARISON BETWEEN PREDICTED AND EXPERIMENTAL HEAT TRANSFER COEFFICIENTS IN NON-NEWTONIAN AND DRAG-REDUCING FLOWS

Previously published heat transfer data of Gupta [19, 20] and Friend [21, 22] will be used in the present study to compare predicted and experimental heat transfer coefficients.

Gupta [19, 20] measured heat transfer coefficients for the flow of 0.01%, 0.05% and 0.45% ET597 partially hydrolyzed polyacrylamide solutions in water. Checks on the heat balance for the experiments agreed within 17% for a test run with water and 8% for 0.01% and 0.05% ET597. The 0.45% solution exhibited transition delay, as illustrated in Fig. 2(b), and will not be considered in this paper.

Gupta's [19] viscometric data are shown in Figs. 3 and 4. It would be expected at the low concentrations of 0.01 and 0.05% the fluids would exhibit essentially Newtonian laminar flow characteristics. The solid lines shown in Figs. 3 and 4 indicate Newtonian behaviour (having a slope of unity) and it can be seen that this represents the data well over large ranges of shear rate. The variation of viscosity with temperature for both fluids has been represented by an exponential function of the form

$$\mu = \mu_1 \exp\{\alpha_1/T\} \tag{17}$$

where μ_1 and α_1 are constants and T is the absolute temperature. This variation of viscosity is shown in Fig. 5. In his original work Gupta [19] represented his viscometric data using the generalized power law of Metzner and Reed [1, 23] and accounted for variations in viscosity by interpolation using the reciprocal of absolute temperature. The friction factor data of Gupta are presented in Fig. 6 which shows that the 0.01% solution gives very little drag reduction and is close to Newtonian behaviour in the turbulent region. However, the 0.05% solution is highly drag reducing in turbulent flow even though it is Newtonian in laminar flows.

Table 2 gives a comparison of Nusselt numbers calculated from the various eddy viscosity expressions and Gupta's experimental data for polymer solutions. Gupta found that his tests with water agreed with standard Newtonian empirical correlations, with a maximum deviation of 15.5%. The values in Table 2 are mean percentage deviations between predicted and experimental Nusselt numbers. Densities, thermal conductivities and heat capacities were assumed to be the same as for water since the polymer concentrations are very low. Physical properties were based on the arithmetic mean bulk temperature of the liquid.

Table 3 shows values again based on the bulk temperature of the fluid but with the predicted Nusselt numbers corrected using the Sieder–Tate correction factor, i.e.

$$Nu_{\text{corrected}} = Nu_{\text{predicted}} \{\mu_b/\mu_w\}^{0.14} \tag{18}$$

From Tables 2 and 3 it can be seen that

(1) In general the agreement between theoretical predictions and experimental measurements is good.

(2) The two alternative methods of calculating the coefficient B from either equation (15) or (16) give results which are comparable in accuracy.

(3) The performance of the various eddy viscosity expressions differs between the two fluids. These liquids differed greatly in their degree of drag reduction, the lower concentration solution being almost Newtonian, as can be seen from the friction factor data shown in Fig. 6. However, the fit of the predictions to the data for the higher concentration solution, with strong drag reduction, is not significantly worse than for the low concentration solution.

(4) The eddy viscosity expression which gave the best overall performance was that of Mizushina and Ogino [14].

Table 1. Summary of the equations used to describe the eddy viscosity expressions for the calculation of dimensionless heat transfer coefficients

Eddy viscosity	0–y_1^+	y_1^+–y_2^+	y_2^+–R^+
[12]	6	6	11
[13]	8	10	11
[14]	9	10	11
[15]	13	10	11

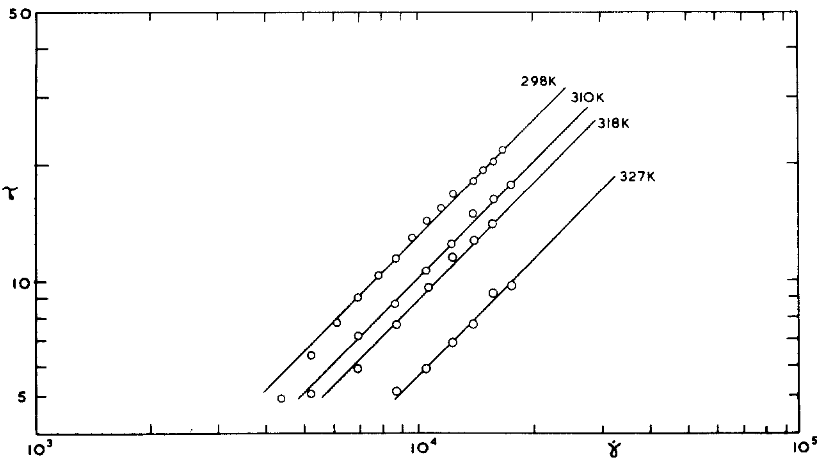

FIG. 3. Gupta's viscometric data for 0.01% ET597.

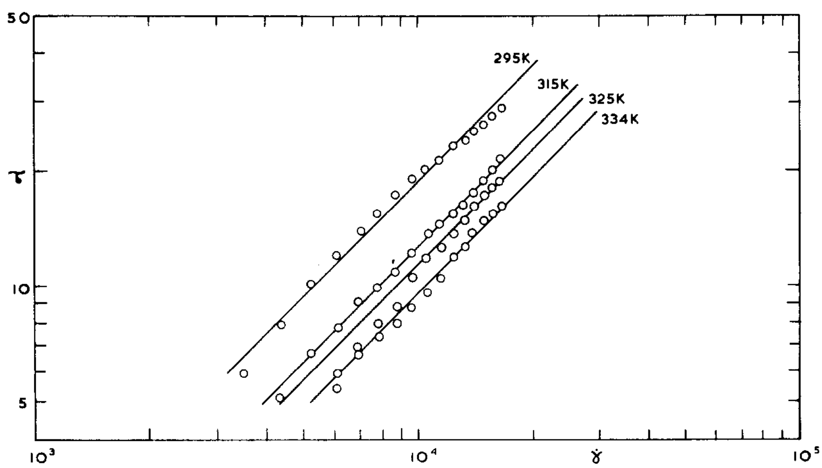

FIG. 4. Gupta's viscometric data for 0.05% ET597.

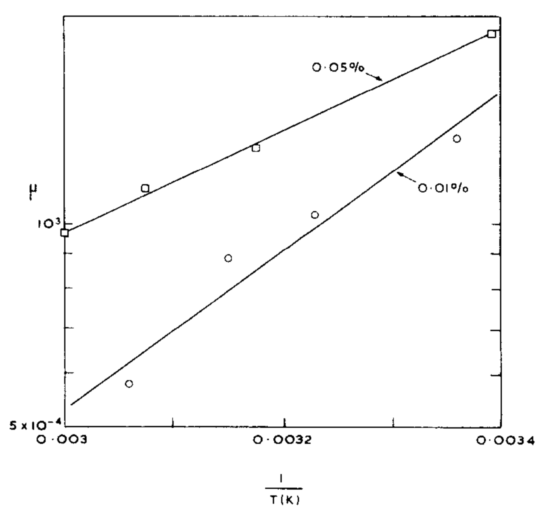

FIG. 5. Temperature dependence of Gupta's viscometric data.

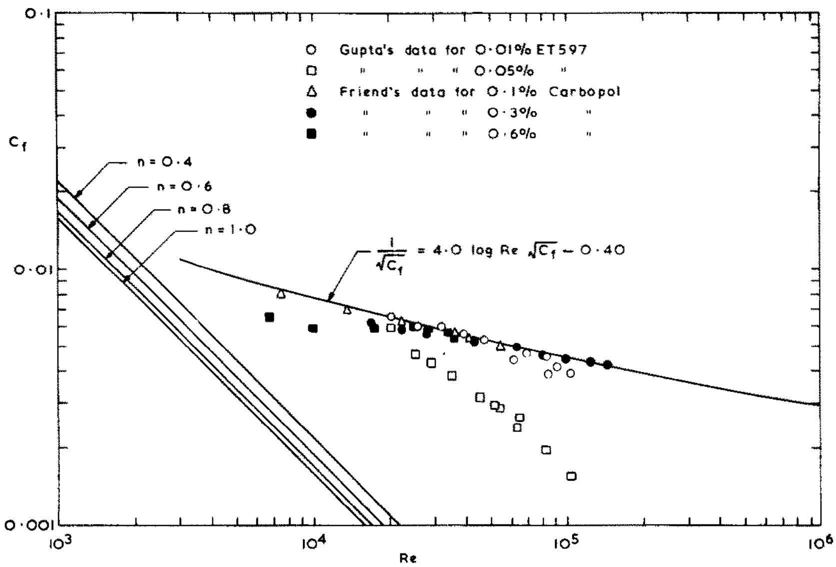

FIG. 6. Friction factor data of Gupta and Friend.

Table 2. Mean percentage deviations between predicted and experimental Nusselt numbers for the heat transfer data of Gupta [19]

	0.01% ET597		0.05% ET597	
Eddy viscosity expression	B coefficient from [10]	B coefficient from [17]	B coefficient from [10]	B coefficient from [17]
[12]	8.9	8.0	18.0	20.5
[13]	20.3	19.4	13.2	14.1
[14]	15.5	14.6	10.8	13.1
[15]	8.2	7.3	16.1	17.8

Table 3. Mean percentage deviations between predicted and experimental Nusselt numbers for the heat transfer data of Gupta [19] with predicted Nusselt numbers corrected by the Sieder–Tate correction

	0.01% ET597		0.05% ET597	
Eddy viscosity expression	B coefficient from [10]	B coefficient from [17]	B coefficient from [10]	B coefficient from [17]
[12]	6.9	5.9	19.2	21.9
[13]	16.4	15.4	12.8	14.9
[14]	11.5	10.7	12.4	14.8
[15]	6.5	5.5	20.8	22.5

(5) The Sieder–Tate correction improved predictions for the 0.01% fluid which showed the lower degree of drag-reduction but had a detrimental effect on the predictions for the 0.05% fluid.

The experimental data of Friend [21] will now be compared with theoretical predictions. Friend measured heat transfer coefficients for the turbulent flow of solutions of carbopol (carboxypolymethylene) in water. The accuracy of their data was such that the heat balance checked within 15%. Most investigators who have studied carbopol solutions have assumed that they would not exhibit drag-reduction [24, 25], but as has previously been shown, this is not always true [1].

The carbopol solutions used by Friend exhibited significantly non-Newtonian viscous characteristics. Figure 7 illustrates Friend's viscometric data. It can be seen that the data approximate well to the power law behaviour

$$\tau = K\dot{\gamma}^n. \tag{19}$$

Thus, these carbopol solutions are non-Newtonian, shear thinning liquids which in the turbulent region

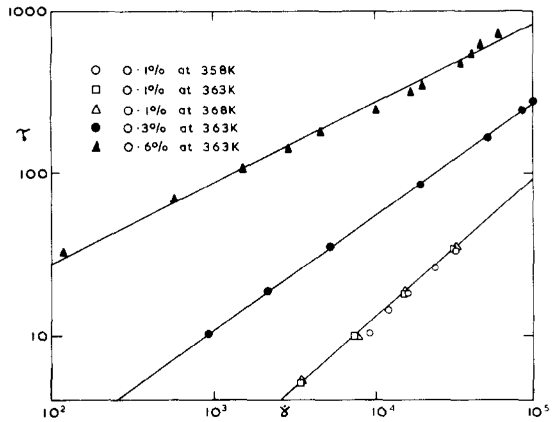

FIG. 7. Friend's viscometric data for carbopol.

Table 5 gives the mean percentage deviations for the four eddy viscosity expressions averaged out over all of the data considered in this investigation. Although the van Driest [15] expression is seen to give the smallest errors overall, it is recommended that the expression of Mizushina and Ogino [14] be used. The errors with Mizushina and Ogino are almost as low as van Driest's but the Mizushina expression gives a far better performance at high levels of drag reduction, as can be seen in Tables 2 and 3. In our investigation only the 0.05% data of Gupta exhibited a significant level of drag reduction.

It is worthy of note that the errors quoted in Tables 2–5 could have been reduced by smoothing out the experimental friction factor data used to calculate the heat transfer coefficients rather than taking the actual measured values. This can be illustrated by considering the data of Friend [21, 22] to be purely viscous and calculating the required friction factors from the Nikuradse equation. (From Fig. 6 it can be seen that most of Friend's data are apparently purely viscous.) Table 6 shows mean percentage deviations calculated on this basis. Comparing the values in Table 4 and Table 6 it can be seen that smoothing the friction factor data does in most cases reduce the errors.

Table 4. Mean percentage deviation between predicted and experimental Nusselt numbers for the heat transfer data of Friend [21]

	0.1% carbopol ($n = 0.87$)		0.3% carbopol ($n = 0.707$)		0.6% carbopol ($n = 0.491$)	
Eddy viscosity expression	B coefficient from [10]	B coefficient from [17]	B coefficient from [10]	B coefficient from [17]	B coefficient from [10]	B coefficient from [17]
[12]	6.9	6.8	16.4	16.2	26.0	26.1
[13]	18.4	18.2	14.5	13.8	17.9	18.0
[14]	14.5	14.3	13.5	13.0	20.9	21.0
[15]	6.8	6.7	15.8	15.7	23.6	23.7

give only slight drag reduction (Fig. 6). Friend treated his data for 0.3 and 0.6% solutions using the generalized approach of Metzner and Reed [23]. He did not measure any significant effect of temperature on the viscous characteristics for the small temperature ranges used in this investigation.

Table 4 gives a comparison between predicted and calculated Nusselt numbers for Friend's carbopol data. The values in Table 4 are mean percentage deviations between predicted and experimental Nusselt numbers. Friend found that test runs with water deviated from standard empirical correlations for Newtonian liquids with a maximum of 9%.

From Table 4 it can be seen that

(1) As with Gupta's data the general agreement between theoretical predictions and experimental measurements is good.

(2) The discrepancy between predictions and measurements increases as the fluid becomes very non-Newtonian. This may be as a result of trying to represent the flow by a single viscosity characteristic of the flow, i.e. the viscosity at the wall. For severe non-Newtonian behaviour it may be necessary to solve the equations of motion and energy simultaneously with the constitutive equation for the non-Newtonian viscosity. However, strong non-Newtonian behaviour is seldom a problem with turbulent flow in practice as it is usually associated with high viscosity and the fluid would be pumped under laminar flow conditions.

CONCLUSIONS

(1) It has been demonstrated that four previously published Newtonian eddy viscosity correlations can be successfully adapted to predict heat transfer coefficients in both non-Newtonian and drag-reducing turbulent pipe flow.

(2) Comparison between the previously published

Table 5. Mean percentage deviations between predicted and experimental Nusselt numbers for the heat transfer data of Gupta [19] and Friend [21] considered in this study

Eddy viscosity expression	B coefficient from [10]	B coefficient from [17]
[12]	15.2	15.3
[13]	16.7	16.5
[14]	14.6	14.8
[15]	14.0	14.2

Table 6. Mean percentage deviations between predicted and experimental Nusselt numbers for the heat transfer data of Friend [21] and friction factors assumed to be purely viscous

Eddy viscosity expression	0.1% carbopol ($n = 0.87$)	0.3% carbopol ($n = 0.707$)	0.6% carbopol ($n = 0.491$)
[12]	6.2	17.2	20.5
[13]	16.1	10.3	10.7
[14]	12.1	11.6	14.3
[15]	6.7	16.3	19.0

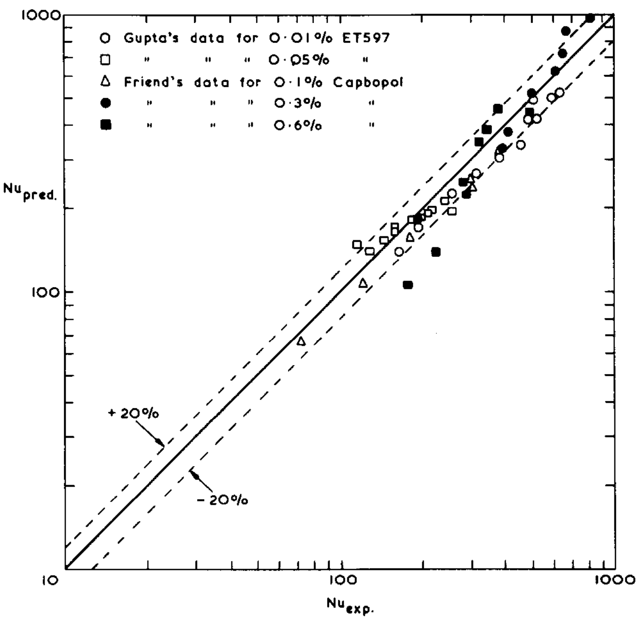

FIG. 8. Predicted vs experimental Nusselt numbers using the eddy viscosity correlation of Mizushina and Ogino [14] with the B coefficient calculated from Edwards and Smith [10].

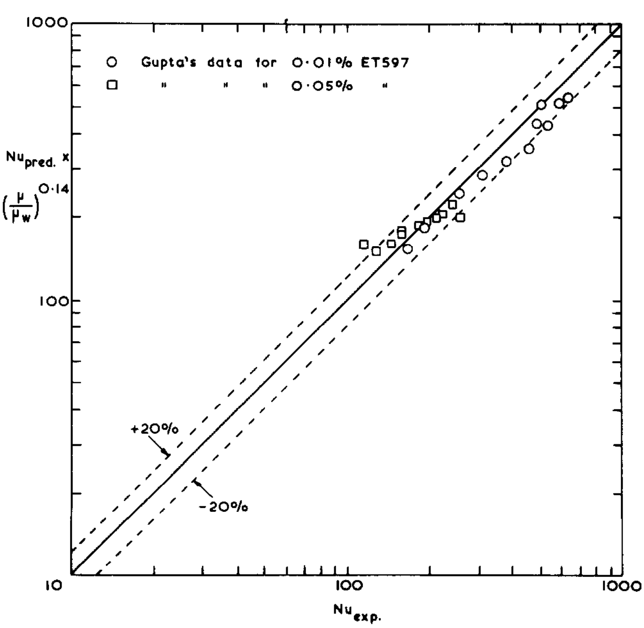

FIG. 9. Predicted vs experimental Nusselt numbers for Gupta's data using the eddy viscosity correlation of Mizushina and Ogino [14] with the B coefficient from Edwards and Smith [10] and applying the Sieder–Tate correction factor.

experimental data of Gupta [19] and Friend [21] and theoretical predictions show good agreement over the Reynolds number range of 6×10^3–1.5×10^5 (Figs. 8 and 9). No systematic deviations between experiment and theory are observed.

(3) Over the range of experimental data examined the eddy viscosity expressions of Mizushina and Ogino [14] and van Driest [15] show the best agreement.

(4) The expression of Mizushina and Ogino is recommended for general use rather than the van Driest expression because the former shows much better performance with drag-reducing fluids.

(5) The expression of Edwards and Smith and Arunachelem, Hummel and Smith [17] for the calculation of the B coefficient for drag-reducing fluids give similar predictions.

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APPENDIX

The effective radial flux is given by

$$q_{r\,\mathrm{eff}} = (k + \rho C_p \varepsilon_{hr}) \frac{\partial \bar{T}}{\partial y}. \tag{A1}$$

From the equation of motion the analogous expression to equation (A1) is

$$\bar{\tau}_{rz\,\mathrm{eff}} = (\mu + \rho \varepsilon_{rz}) \frac{\partial \bar{v}_z}{\partial y}. \tag{A2}$$

If we assume the radial flux to vary linearly [10, 26] then we can combine equations (A1) and (A2) to give

$$\frac{\partial \bar{T}^+}{\partial \bar{v}_z^+} = \left(1 + \frac{\rho \varepsilon_{rz}}{\mu}\right) \bigg/ \left(\frac{1}{Pr} + \frac{\rho \varepsilon_{hr}}{\mu}\right). \tag{A3}$$

Integrating equation (A3) from the wall to the centreline and introducing the turbulent Prandtl number Pr_{tr} gives

$$\int_0^{\bar{T}_R^+} \mathrm{d}\bar{T}^+ = \int_0^{\bar{v}_{z_0}^+} \left[(Pr + Pr_{tr}) \bigg/ \left(1 + \frac{Pr\,\rho \varepsilon_{rz}}{Pr_{tr}\,\mu}\right)\right] \mathrm{d}\bar{v}_z^+ + \int_0^{\bar{v}_{z_0}^+} Pr_{tr}\,\mathrm{d}\bar{v}_z^+ \tag{A4}$$

where

$$\bar{T}_{R^+}^+ = \frac{\bar{T}_{R^+} - T_w}{(q_w/\rho C_p v^*)} = \frac{\sqrt{(C_f/2)}}{C_{h_0}}.$$

Rearranging

$$C_{h_0} = \sqrt{(C_f/2)} \bigg/ \int_0^{\bar{v}_{z_0}^+} \frac{(Pr - Pr_{tr})\,\mathrm{d}\bar{v}_z^+}{(1 + (Pr/Pr_{tr})(\rho \varepsilon_{rz}/\mu))} + \int_0^{\bar{v}_{z_0}^+} Pr_{tr}\,\mathrm{d}\bar{v}_z^+. \tag{A5}$$

The basis of this coefficient, C_{h_0}, can be transferred to the bulk temperature using a simple expression developed by Kader and Yaglom [11]. These authors suggested an approximate relationship between the dimensionless heat transfer coefficient based on the centreline temperature and that based on the bulk temperature, i.e.

$$C_h = C_{h_0} \bigg/ 1 - \frac{C_{h_0} \beta}{\sqrt{(C_f/2)}} \tag{A6}$$

where β is a constant. Equation (A6) is based on the logarithmic temperature profile relationship

$$\frac{\bar{T}_0 - \bar{T}}{T^*} = -\alpha \ln(y/R) + \beta_1 \quad \text{(A7)}$$

where

$$T^* = \frac{q_w}{\rho C_p v^*}$$

and α and β_1 are constants. The constant β in equation (A6) was derived by Kader and Yaglom to be 1.5α. If we assume equation (A7) to be analogous to the corresponding logarithmic equation for velocity profiles, then α would be expected to be 2.5 and hence β to be 3.75. A value of 3.75 is used throughout this work. A slightly different value was used by Kader and Yaglom in their original work.

Combining equations (A5) and (A6) gives

$$C_h = \sqrt{(C_f/2)} \Bigg/ \left[(Pr - Pr_{tr}) \int_0^{\bar{v}_{zo}^+} \frac{d\bar{v}_z^+}{\left(1 + \frac{Pr}{Pr_{tr}} \frac{\rho \varepsilon_{rz}}{\mu}\right)} + Pr_{tr} \bar{v}_{zo}^+ - \beta \right]. \quad \text{(A8)}$$

Goldstein [27] has given a relationship between maximum and bulk velocities, i.e.

$$\bar{v}_{zo}^+ = \frac{4.07\sqrt{(C_f/2)} + 1}{\sqrt{(C_f/2)}} \quad \text{(A9)}$$

Substituting equation (A9) into equation (A8) gives

$$C_h = \sqrt{(C_f/2)} \Bigg/ \left[(Pr - Pr_{tr}) \int_0^{\bar{v}_{zo}^+} \frac{d\bar{v}_z^+}{(1 + (Pr/Pr_{tr})(\rho \varepsilon_{rz}/\mu))} + \frac{Pr_{tr}}{\sqrt{(C_f/2)}} (4.07\sqrt{(C_f/2)} + 1) - \beta \right]. \quad \text{(A10)}$$

This equation can be solved for the dimensionless heat transfer coefficient if the eddy viscosity is known as a function of the velocity. A slightly less accurate form was proposed by Reichardt [26].

TRANSFERT THERMIQUE DANS UN ECOULEMENT TURBULENT EN CONDUITE DE FLUIDES NON-NEWTONIENS A REDUCTION DE FROTTEMENT

Résumé—Une approche théorique basée sur les formulations de viscosité turbulente newtonienne est appliquée au calcul des coefficients de transfert pour des liquides non-newtoniens à réduction de frottement, dans un écoulement turbulent pleinement développé en conduite. Des données antérieures de Gupta [19] et Friend [21] pour des nombres de Reynolds entre 6×10^3 et $1,5 \times 10^5$ sont comparées avec les valeurs calculées et on obtient un bon accord.

WÄRMEÜBERGANG AN NICHT-NEWTON'SCHE WIDERSTANDSVERMINDERNDE FLUIDE BEI TURBULENTER ROHRSTRÖMUNG

Zusammenfassung—Es wurde eine theoretische Untersuchung auf der Grundlage der newton'schen Schein-Viskositätsbeziehungen gemacht, um die Wärmeübergangskoeffizienten bei voll ausgebildeter turbulenter Rohrströmung von nicht-newton'schen widerstandsvermindernden Fluiden zu berechnen. Zum Vergleich wurden die Daten von Gupta [19] und Friend [21] herangezogen, die vor kurzem veröffentlicht wurden. Die Reynolds-Zahlen lagen im Bereich von 6×10^3 bis $1,5 \times 10^5$. Es zeigte sich gute Übereinstimmung.

ТЕПЛОПЕРЕНОС К НЕНЬЮТОНОВСКИМ И СНИЖАЮЩИМ СОПРОТИВЛЕНИЕ ЖИДКОСТЯМ ПРИ ТУРБУЛЕНТНОМ ТЕЧЕНИИ В ТРУБЕ

Аннотация — Ньютоновские соотношения для коэффициента турбулентного перемешивания использованы для расчета коэффициентов теплообмена неньютоновских и снижающих сопротивление жидкостей при полностью развитом турбулентном течении в трубе. Полученные значения сравнивались с ранее опубликованными данными Гупты и Френда в диапазоне значений числа Рейнольдса от $6 \cdot 10^3$ до $1,5 \cdot 10^5$ и получено хорошее соответствие.